CALCULATION OF FRACTURE DIAGRAMS

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This article discusses equations describing critical and subcritical fracture diagrams, obtained from the energy criterion for fracture in an integral formulation. The equations take approximate account of the presence of a small plastic region ahead of the end of a crack and include the coefficient of the intensity of the stresses and, in the case of cyclic loading, one empirical coefficient. The results of the calculation are in agreement with experiment.

A functional dependence between the external loading and the length of the principal fracture in a flat sample, called the fracture diagram, reflects the ability of a material to resist the propagation of a fracture, and can possibly serve as a basis for the design calculations of structural parts.

Fracture diagrams are determined experimentally using apparatus which records the length of a fracture [1]. Fracture diagrams can be obtained by calculation on the basis of an assumed model of the fracture into whose formulation a small number of experimentally determined characteristics of the material enter [2, 3].

We shall examine one possible method of making such a calculation and compare its results with experiment.

We write the energy criterion for a fracture with a thin plastic zone ahead of its edge [3]. For definiteness we shall consider a two-dimensional body with a single rectilinear fracture ($y=0$, $|x| \le l$)

$$\frac{\partial}{\partial l}\int_0^l (2\gamma - \sigma_y v)\,dx - \frac{\partial}{\partial l}\int_l^a (\sigma_y - \sigma_0)\,v\,dx = 0. \tag{1}$$

Here $\sigma_y = \sigma_y(x)$ is the stress from a given loading, arising at the x axis in a body without a fracture. This stress enters into Eq. (1) with an opposite sign. Displacement of the points of the surface of the section $v = v(x, l)$ develops in the direction of the y axis as a result of the action of the cracking stress σ_y in the region $y=0$, $|x| \le l$, and of the stress $\sigma_y - \sigma_0$ in the regions $l < |x| \le a$. The plastic zone occupies the region $y = l < |x| \le a$, which, for the solution of the problem, is taken as the section. The stress σ_0 is symmetrically applied to the surface of the section $l < |x| \le a$ and, furthermore, we assume that the limit of strength of material, σ_b, is constant. The surface density of the fracture energy is equal to γ.

After differentiation of Eq. (1) we obtain

$$2\gamma - \int_0^l \sigma_y \frac{\partial v}{\partial l}\,dx + \sigma_0 v(l, l) - \int_l^a (\sigma_y - \sigma_0)\frac{\partial v}{\partial l}\,dx = 0. \tag{2}$$

To take approximate account of the presence of a plastic zone we assume the condition of the smallness of its length, $a - l$; this means that in the integral, taken within the limits from l to a, we shall assume that $\sigma_y(x) = \sigma_y(l)$. In addition, we postulate the invariability of the form of the plastic zone, which is expressed by the condition for self-similarity

$$\frac{\partial v}{\partial l} = -\frac{\partial v}{\partial x}.$$

Taking into consideration that $v(a, l) = 0$, we find

$$\int_l^a (\sigma_y - \sigma_0)\frac{\partial v}{\partial l}\,dx = [\sigma_y(l) - \sigma_0]\,v(l, l).$$

Substitution of this integral into (2) gives the relationship

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$$2\gamma - \sigma_y(l)\, v(l, l) - \int_0^l \sigma_y \frac{\partial v}{\partial l}\, dx = 0 \tag{3}$$

If, in this equation, the derivative $\partial v/\partial l$ is determined from the elastic solution, we arrive at an energy criterion of the form

$$\delta \int_0^l (2\gamma - \sigma_y v)\, dx = 0 \tag{4}$$

in which the postulation $v(l, l) \neq 0$ approximately reflects the existence of a plastic zone at the end of the section.

Condition (4) serves to calculate: a) the critical fracture diagram with

$$\delta = \frac{\partial}{\partial l}\, \delta l$$

and b) the subcritical fracture diagram with

$$\delta = \left(\frac{\partial}{\partial l} + \frac{\partial}{\partial p}\frac{dp}{dl}\right) \delta l$$

where p is the parameter of the external loading ($\sigma_y(x) = pf(x)$).

Further, taking account of the known relationship

$$K^2 / E = \int_0^l \sigma_y (\partial v / \partial l)\, dx$$

and assuming [4] that

$$\sigma_y(l)\, v(l) = 2\gamma \sigma_y^2(l) / \sigma_b^2$$

we obtain

for case a)

$$2\gamma\, [1 - \sigma_y^2(l) / \sigma_b^2] - K^2 / E = 0 \tag{5}$$

for case b)

$$\frac{dp}{dl} = \frac{2\gamma\, [1 - \sigma_y^2(l) / \sigma_b^2] - K^2 / E}{(2 / E) \int K (\partial K / \partial p)\, dl + 4\gamma l \sigma_y(l) / \sigma_b^2} \tag{6}$$

In dimensionless form, Eqs. (5) and (6), respectively, have the form (here $\sigma_y(x) \equiv \text{const} = p$)

$$1 - \lambda^2 - K_0^2 = 0 \tag{7}$$

$$\frac{d\lambda}{d\zeta} = \frac{1}{2} \frac{1 - \lambda^2 - K_0^2}{\lambda\zeta + \int K_0 (\partial K_0 / \partial \lambda)\, d\zeta} \tag{8}$$

$$(\lambda = p / \sigma_b,\ \zeta = l / c,\ K_0 = K / K_c,\ c = \pi K_c^2 / 8\sigma_b^2)$$

where K_c is the critical coefficient of the intensity of the stresses.

These equations take account of the form of the body and of the scheme of loading, by the coefficient of the intensity of the stresses, K.

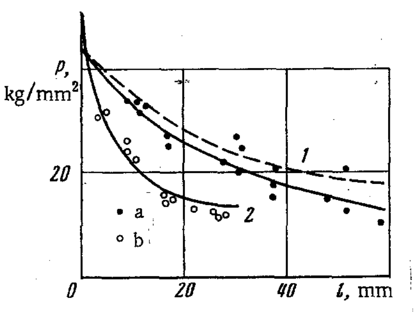

Fig. 1

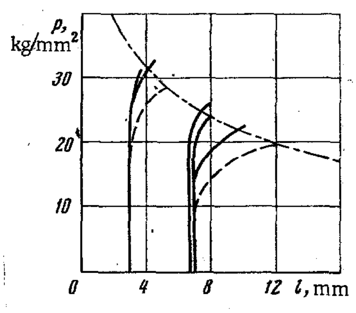

Fig. 2

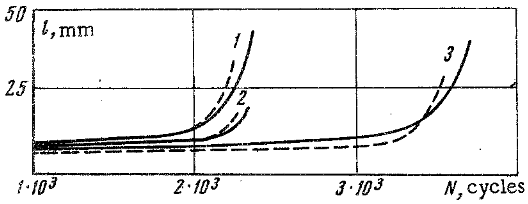

Fig. 3

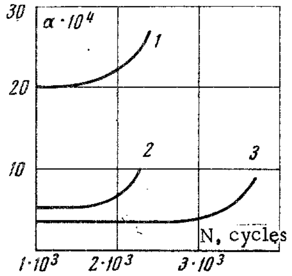

Fig. 4

For the extension of a band of width $2b$ with a central fracture of length $2l$ we have ($\beta = b/c$) [5]

$$K_0 = \pi\lambda \sqrt{\zeta/8}\,[1 + 0.595\,(\zeta/\beta)^2 + 0.481\,(\zeta/\beta)^4]\,. \tag{9}$$

Critical fracture diagrams, plotted using Eq. (7) taking expression (9) into account, are shown by the solid lines on Fig. 1. The calculations were made for flat samples made of aluminum alloys D16T-1, points a, curve 1 ($\sigma_b = 44.6$ kg/mm^2, $E = 7\times10^3$ kg/mm^2, $K_c = 252$ kg/mm$^{3/2}$) with a size of $600\times200\times1.4$ mm^3, and VAD-23, points b, curve 2 ($_b = 49.7$ kg/mm^2, $E = 7.3\times10^3$ kg/mm^2, $K_c = 125$ kg/m$^{3/2}$) with a size of $300\times100\times1.8$ mm^3. The experimental points are plotted on the same figure, each point corresponding to one sample. The dotted line shows the critical diagram for a plane, obtained for alloy D16T-1 from the energy criterion, taking account of the thin plastic zone ahead of the end of the fracture [3]. For VAD-23, the solutions for a band and a plane practically coincide.

Figure 2 gives a comparison of subcritical fracture diagrams, obtained from experiment and plotted using Eq. (8), for alloy VAD-23; the solid lines are experimental, and the dotted lines calculated.

Equation (6) or (8) also makes it possible to calculate the increase in the length of a fracture with the number of cycles, N, under repeated variable loading. To this end, a family of integral curves $p(l)$ of Eq. (6) was plotted, whose parameters are the initial length of the fracture. Each cycle of the loading, when the loading parameter is varied within the limits from p_{min} to p_{max}, corresponds to an increment in the length of the fracture Δl, determined from the integral curve $p(l)$ [3]. The service life with respect to the number of cycles is determined by the condition $dp/dl = 0$, i.e., the calculation is terminated at the moment when the length of the fracture attains a critical value with $p_{min} < p \leq p_{max}$.

Figure 3 shows the results of calculation and experiment with repeating static loading for sheet samples of the previous sizes made of D16T-1 and SAP ($\sigma_b = 32.5$ kg/mm^2, $E = 7\times10^3$ kg/mm^2, $K_c = 245$ kg/mm$^{3/2}$) and sheet samples cut out in the direction of rolling, with a size of $300\times100\times1.5$ mm^3, made of titanium alloy VT-14 ($\sigma_b = 130$ kg/mm^2, $F = 11.5\times10^3$ kg/mm^2, $K_c = 200$ kg/mm$^{3/2}$). The initial length of the fracture in the aluminum alloys $2l_0 = 12$ mm, the maximal stress of a cycle $p_{max} = 16$ kg/mm^2, the coefficient of the asymmetry $r = 0.2$, and the frequency 200 cycles/min. For the titanium alloy, $2l_0 = 10$ mm, $p_{max} = 26$ kg/mm^2, $r = 0.2$. The solid lines represent calculation, and the dotted lines the result of experiment: 1) D16T-1; 2) VT-14; 3) SAP.

In calculating the curves $l - N$, the partial decrease in the length of a fracture with loading was taken into consideration. This experimentally known effect [6] is obviously due to residual compressive stresses arising with removal of the load in the region of plastic deformations at the end of the fracture. The opening-up of a fracture was taken into account by the coefficient α, which was introduced in the following manner: $l_{i+1} = l_i + \alpha\Delta l$, where l_{i+1} is the length of the fracture before the i+1st cycle, and Δl is the increment in the length of the fracture in the i-th cycle. It was established by trial and error that the coefficient α varies as a function of the number of cycles in accordance with the law represented in Fig. 4. The legend is the same as in Fig. 3. This dependence can be assumed to be characteristic for a given material and it is proposed to use it in calculation of the service life of structural elements whose form differs from the form of the sample.

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